Making Connections: Technology-Based Science Experiments for Teaching and Learning Mathematics

Irina Lyublinskaya
College of Staten Island, Staten Island, NY, USA
Lyublinskaya@mail.csi.cuny.edu

Abstract

Using science experiments in different courses of mathematics helps ground students' understanding of abstract mathematics concepts in real-world applications. Hands-on activities connect mathematics with science in a way that is accessible to teachers and students alike. Suggested experiments are designed for students taking different courses of mathematics from Algebra to Calculus. In addition, these experiments expose students to different roles of mathematics in science, usage of different mathematical techniques to verify results of experiments, applications of mathematical modeling, and development conjectures based on the results of the experiment that go beyond the scope of the experiment. This hands-on approach also allows students to use technology and different measuring equipment in mathematics classes. Real-life problems do not provide us with "nice" numbers. Students educated on sets of standard problems get accustomed to the fact that only "nice" numbers, usually integers, can be correct answers to the problem. Real practical problems give students an understanding that a number that is not very "nice" can be a correct answer to the problem. In addition to graphing calculators, experiments use the data interfaces such as Texas Instruments Calculator Based Laboratory, CBL2™, Vernier LabPro™, or Vernier EasyLink™ with different probes such as common science equipment, and basic tools. Described examples of such activities connect mathematics with science in a way that is accessible to teachers and students alike. Each activity explores a scientific phenomenon and connects it to mathematics concepts such as linear modeling, properties of cosine graph, and parametric differentiation.

Introduction

Teachers everywhere are constantly facing the challenge of making mathematics meaningful for all their students. Studies show that real-life applications, especially visual and hands-on demonstrations, enhance students' learning of the material, meet needs of students with different learning styles, and create additional motivation for learning a discipline [Cortes-Figueroa & Moore, 1999; Niess, 2001; Stager, 2000]. The use of science experiments allows students to create visual image and practical understanding of abstract mathematics concepts and relationships. Experimental demonstrations and lab activities that use technology in the course of mathematics make mathematics more interesting and appealing to students.

Using science experiments in teaching mathematics helps students to realize that mathematics plays an important role in every aspect of their lives, especially in science applications. Mathematics is needed at each step of scientific investigation. In high school science curriculum students are usually exposed to only one role of mathematics – use of mathematical technique for computations of parameters from the experimental results or verification of experimental and theoretical data. In students' minds this approach reduces the role of mathematics to a basic computational tool. By using science experiments in mathematics classes, students are also exposed to other roles of mathematics in science:

- The use of a set of mathematical models and methods that allow them to describe some real-life situation, and to design an experiment for this situation, and

- The development of conjectures based on the results of the experiment that go beyond the scope of the experiment; only mathematics allows verification of these conjectures for general or extreme cases.

Classroom experience demonstrates that use of hands-on activities within rigorous mathematics content provides additional opportunities for students to make connections, and to master concepts and skills [Lyublinskaya, 2003a&b]. Real-life problems do not provide students with "nice" numbers. Students educated on sets of standard problems get accustomed to the fact that only "nice" numbers, usually integers, can be correct answers to the problem. Real practical problems give students an understanding that a number that is not very "nice" can be a correct answer to the problem.

The following three examples illustrate how technology-based science experiments could be used to engage students in meaningful classroom activities while teaching them rigorous mathematics concepts and skills.

Linear Modeling – The Case of Vandalism

"It was a shocking morning for many students and instructors as they arrived at Mountain Community College. It seems that the night before vandals had used black paint to scrawl several offensive phrases and pictures on the front of the academic building and sidewalk leading to the main entrance of the school. Footprints were found leaving the scene of the crime (down the sidewalk away from the main entrance – see Figure 1). The footprints originated in a puddle of spilled paint and extended some ten strides from the origin. After examining the footprints left at the scene and comparing them with several suspects, your team should be prepared to present quantitative evidence to the local police department that will allow them to obtain a warrant and possibly make an arrest." Students then are given three suspects with their motives and heights. Click here for complete scenario and lab handout.

Figure 1. Stride Pattern Left at the Crime Scene

This problem is appropriate for different college algebra courses. Students get immediately engaged in this forensics problem. They are asked to investigate the relationship between the stride pattern left at the crime scene and different parameters such as height of a person walking and running, speed of walking or running, and any other parameters that students may think are important. Based on their investigation, students should develop quantitative method of analysis that would give them sufficient evidence to choose one of the suspects from the given list. By using the CBR2™ (Calculator Based Ranger) with TI-84 graphing calculator, students can plot the distance-time graph of a walking or running person (see Figure 2).

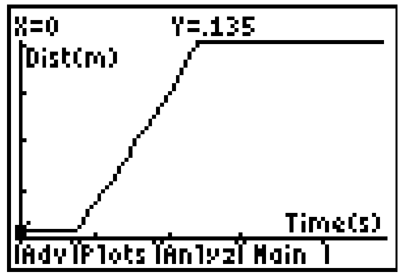
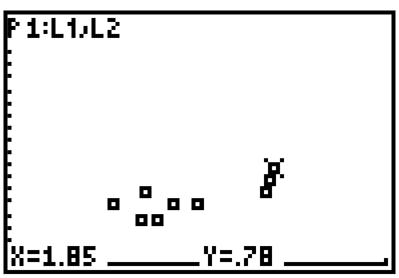
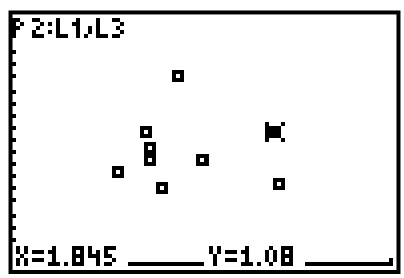


Figure 2. TI-84 Calculator Screen for Distance-Time Graph of a Walking Person

Students have to analyze the piece-wise function produced by walking and decide what each segment represents. They select the region where the function has a positive slope, since that piece represents the walking. From this graph they can then determine total distance walked by a person as the change in the value of the function, they can also determine the average stride length, as that change divided by the number of walked strides, and then they can determine the speed of walking as the slope of the line. In addition, each student records his or her own height. Data is collected from all students in class and plotted on the same scatter plot. Sample scatter plots of height vs. average stride length for a group of students walking and running at a constant pace are shown at Figure 3.



Walking at a constant pace



Running at a constant pace

Figure 3. Scatter Plot of Height vs. Average Stride Length

After the class has collected this information and all data is shared, students start discussion of the following questions:

- What is the relationship between walking/running speed and average stride distance?
- What is the relationship between a person's height and their stride distance during walking? and during running? Can you determine a mathematical model of this relationship?

- Based on your analysis of the experiment, can you determine the height of the perpetrator, and if he/she was running or walking while leaving the crime scene?

In order to answer these questions students have to create a mathematical model for the collective data sample, interpret this model and use it in order to determine the height of the suspect based on the stride distance. Linear regression is used in this case as a realistic approximation of the situation. The steps of an analysis process with the TI-84 graphing calculator are illustrated using TI-SmartView software script [Texas Instruments]. The software package is required in order to run this script. For the readers who do not have access to the TI-SmartView software package, TI-84 graphing calculator key stroke sequence is provided here. For more challenging problem, the stride pattern left at the crime scene could consist of two pieces itself – walking and running (the perpetrator walking away from the scene could be startled by noise and starts running). In this case students will have to use both of the linear models they created. This more challenging situation is appropriate for advanced level college algebra or pre-calculus classes, since it will involve additional investigation of the datasets of the stride length vs. the height collected in the experiment.

This relatively simple activity provides an opportunity for the students to engage in exploration and analysis while learning at a deeper conceptual level a wide range of important mathematics topics and concepts. These concepts would incorporate distance, speed, acceleration, average speed, average distance, scatter plot, regression, best-fit curve, residuals, slope-intercept form of linear equation, range and domain of a function, piece-wise function, interpolation of data, linear modeling, and measurements.

Properties of Cosine Graph – Thrust Force Experiment

"In a *Die Another Day* James Bond is in a fast-paced hovercraft chase. Hovercraft is a ground or water-effect vehicle. There is very little friction between the craft and the surface. Like the hovercraft, the PASCO [PASCO Scientific] fan cart that students are using in this experiment is powered by the airflow created by the fan mounted on top of the cart (Figure 4).

Figure 4. PASCO Fan Cart

The airflow produced by the fan creates a force F acting on the cart in the direction opposite to the airflow that causes the cart to move. Since the fan cart is designed to move in one direction only, the thrust force of the fan cart is the component of the force F parallel to the wheels of the cart. This component can be defined as $F_x = F \cos \theta$, where θ is the angle between the cart's direction of motion and the direction of the force F (Figure 5).

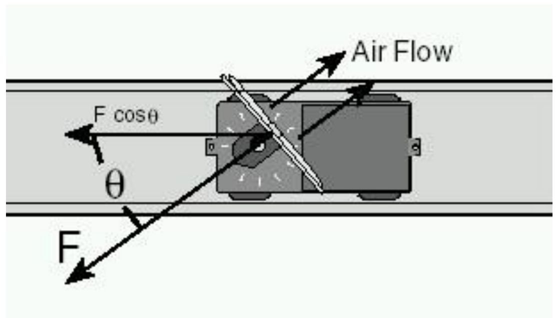


Figure 5. Force Diagram for the Fan Cart

By turning the plane of the fan students can change the direction of airflow and observe the effect of the angle θ on the thrust force F_x. In this experiment students investigate the graph of the cosine function by measuring the thrust force as a function of the angle. This experiment is essentially a hands-on technology-based version of unit circle analysis. This activity can be used in precalculus/trigonometry courses. Click here for the student lab handout developed to be used with the Vernier LabPro™ interface [Vernier Software and Technology] and TI-84 graphing calculator.

Before beginning the experiment, students are asked to predict when the thrust force will be maximal and minimal. Most of the students can easily predict when the thrust force is parallel to the wheel axes of the fan cart, the cart go fastest, and of course they can easily check that by turning the switch on and letting the cart go. It is not as obvious for them what happens when the fan is turned perpendicular to the wheel axes. An immediate check demonstrates that the cart does not go anywhere, since the thrust force pushes the cart perpendicular to the direction the cart could go. With the use of Vernier Dual Force Sensor, data interface (Vernier LabPro™ or CBL2™ or EasyLink™ also available from Vernier Software and Technology), and TI-84 graphing calculator data for all different positions of the fan dial can be collected. The force sensor allows measuring an average force for each dial position. Data then can be recorded and displayed graphically as a scatter plot. The calculator screen shot for the sample data for this experiment is shown on Figure 6.

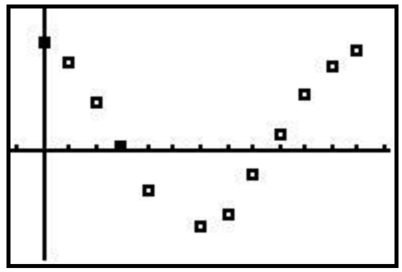


Figure 6. Experimental Scatter Plot for the Fan Cart Activity

Students can now analyze the graph and answer questions about cosine graph properties, for example:

1. At what angle(s) the magnitude of the thrust force is zero? Why?
   Expected Answer: θ = 90° and 270°. Students could provide physical or mathematical explanations, such as air blows perpendicular to the wheel axel so the fan cart does not move, there is no thrust force on the cart, or thrust force is defined as *x* – component of the air flow force, $F_x=F\cos\theta$, so since cos 90° = 0 and cos 270° = 0, then thrust force is 0 at these angles.
2. At what angles does the thrust force reach its maximum possible magnitude (equal to the air flow force *F*)? Why?
   Expected Answer: θ = 0°, 180°, and 360°. Again, students could provide physical and mathematical explanations. From physical point of view, the maximum possible magnitude of the thrust force is reached when the fan is parallel to the wheel axes or from mathematical point of view when cosine is equal to ±1, which happens at 0°, 180°, and 360°.
3. What is the function that describes the ratio F_x/F? Does it depend on actual values of F_x and *F*?
   The question 3 requires students to derive the equation of a cosine function that best fit the experimental points. Thus, they need to find the amplitude, period, and horizontal shift of the function. Due to the experimental nature of the data (the batteries are getting exhausted), there is a slight damping in the amplitude, so at $x = 0$, $y = 1$, but at $x = 6.28$, $y = 0.91$. The average maximum then is 1.91/2 = .955. Students should also notice that the minimum is reached at $x = 3.14$ and the value of the function is $y = -.71$, which indicates the fact that there is a vertical upward shift of the function. Thus, the most reasonable way to find the amplitude of the function would be to find it as the half distance between the average maximum and the minimum: (.955+0.71)/2 = 0.83. The period of the function is 6.28. The horizontal shift is zero, so the equation is $y = 0.83\cos x$.
4. What are the domain and the range of this function respectively? What are the *x* and the *y* – intercepts?
5. What will happen to this function if we keep turning the fan dial through several revolutions?

One of the advantages of this experiment is the fact that the dual force sensor can measure both pull and push. When the airflow is directed backwards, the force sensor measures negative force. Ask students "What is the meaning of the negative part of the graph? And what does it tell us about magnitude and direction of the thrust force?" This question creates a very rich discussion in class and helps students to make connections between hands-on experience of how fan cart move and increasing and decreasing behavior of a cosine graph.

The activity also opens several opportunities for further explorations. For example, you may ask students to convert degrees into radians and find sine regression for the experimental scatter plot. For the sample data presented on Figure 6, the regression equation is $y = 0.83 \sin(.96x + 1.74)$. The TI-SmartView script of finding sine regression for the experimental data is provided here. For the readers who do not have access to the TI-SmartView software package, TI-84 graphing calculator key stroke sequence is provided here. Ask students to plot sine regression equation along with the function $y = 0.83 \cos(0.96x)$ on the interval $[0, 2\pi]$. Do these functions describe the same graphs? Is it possible for the same graph to have different equations? Ask students to explain their statements using properties of right triangle and definitions of sine and cosine. Or you may want to ask them the meaning of the horizontal shift of 1.74 in the expression for the sine regression and help them to realize that this just represents the co-function identity: $\sin(x + \pi/2) = \cos x$.

In this experiment, fun and engaging activity for students are combined with rigorous mathematics. The experiment helps them to make connections between "making sense" real-life situation: how does airflow affect the motion of a fan cart, and how that is represented in an abstract mathematical object, such as a cosine graph. Based on my classroom experience with this activity, my students no longer had problems with recalling that cosine is a decreasing function in the 1st quarter period, that it has maximum at zero degrees and zero value at 90°. They would think of a motion of a fan cart and the position of the fan and all of these properties made sense to them.

Can properties of the sine graph be explored using fan cart – absolutely yes! The fan cart also comes with the sail, an attachment that you place on top of the cart. Students can measure the angle of the sail instead of the angle of the thrust force, and they will get sine graph instead of cosine graph.

Parametric Differentiation – Boyle's Law

"Astrophysicists design and build detectors to collect X-rays and gamma-rays from astrophysical objects, and then interpret the data. It might seem like hot air balloons would be a little outside of their area of interest, but actually, since the radiation that they are most interested in observing is absorbed by the Earth's atmosphere, many of the high energy astrophysics experiments are flown on balloons. This way they can get above a substantial fraction of the absorbing atmosphere. The size of the balloon used is determined by the weight of the scientific payload. As the balloon ascends it changes shape and size. Imagine that the balloon is sealed so that no air can escape from the balloon. As the altitude of the balloon increases, the air pressure outside of the balloon decreases. In ideal conditions, when the temperature of the gas remains constant, gas pressure is inverse proportional to the gas volume. This relationship is called Boyle's Law, one of the gas laws that scientists use to design balloon flight experiments."

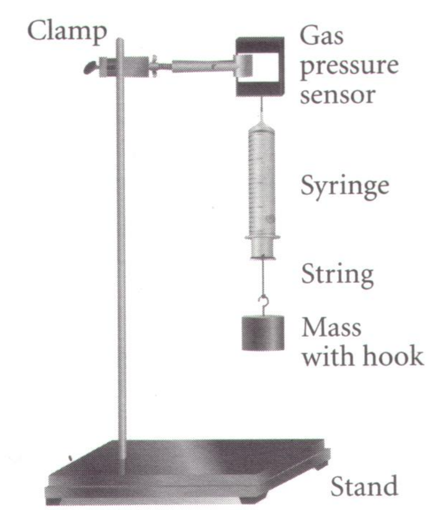


Figure 7. Experimental Setup for Parametric Differentiation Experiment

In the classroom experiment, students will apply Boyle's Law to discover the rule for parametric differentiation. The experimental setup for this activity is shown on Figure 7. While changing mass, *m*, attached to the syringe students will record values of pressure, *P*, measured by Vernier gas pressure sensor, and volume, *V*. Their objective is to explore rates of change of functions $P(m)$, $V(m)$, and $P(V)$, and find relationships between these rates of change.

Students collect data and find the best fit regressions for functions $P(m)$ and $V(m)$. By Boyle's Law, pressure is inverse proportional to the volume if the temperature stays constant, so $pV = \text{constant}$. The sample experimental data is shown in the table below.

Table 1. Sample Experimental Data

Added mass (kg)	Mass, *m* (kg)	Pressure, *P* (kPa)	Volume, *V* (ml)	Reciprocal Volume, 1/V
0	0	103.52	5	.2
1	1	89.217	6	.167
1	2	71.227	7.5	.133
0.5	2.5	60.444	9	.111
0.5	3	51.42	10.5	.095

The calculator screens illustrating scatter plots of experimental data with regression curves are shown on Figure 8.

$P(m) = -17.5m + 105$ $\quad \frac{1}{V}(m) = .035m + .2$ $\quad P(V) = \frac{480}{V}$

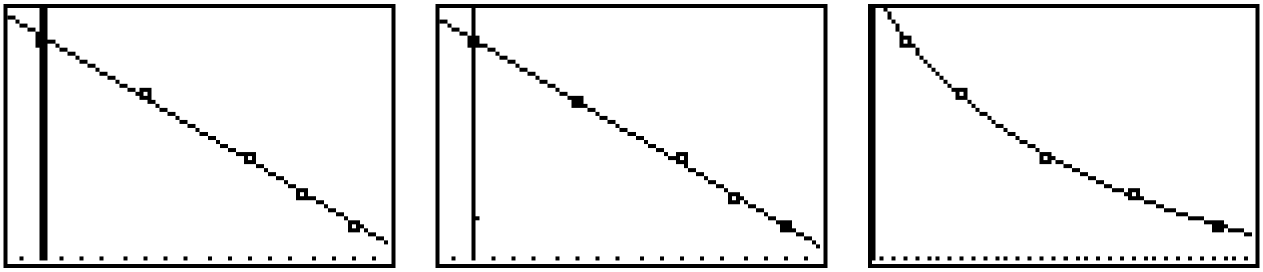


Figure 8. Experimental Scatter Plots with Regression Curves

Students are now expected to complete the following calculations of rates of change:

1. Find derivative of the function $P(m)$: $\frac{dP}{dm} = -17.5$.
2. Solve for $V(m)$: $V(m) = \frac{1}{.035m + .2}$.
3. Take derivative of the function $V(m)$: $\frac{dV}{dm} = -\frac{.035}{(.035m + .2)^2} = -.035V^2$.
4. Find derivative $\frac{dP}{dV}$ directly from $P(V)$: $\frac{dP}{dV} = -\frac{480}{V^2}$.

Students are now expected to compare three derivatives, $\frac{dP}{dm}, \frac{dV}{dm}$, and $\frac{dP}{dV}$, and come up with the rule for parametric differentiation: $\frac{dP}{dV} = \frac{dP/dm}{dV/dm}$.

This activity allows students to discover and verify the rule of parametric differentiation in an engaging chemistry experiment. Besides practicing basic differentiation skills, students also create scatter plots of experimental data and analyze regression equations using residuals. After this hands-on exploration the mathematical proof of the parametric differentiation rule could be expected from the students. See complete lab handout for more details.

Assessment

Science experiments in the mathematics classroom require authentic assessment strategies. One of the most important goals of assessment is to make it a learning tool for the students. If students know how to prepare for the laboratory experiment, what they need before they come to class on the day of the experiment, and if they know how their lab reports will be assessed, they will do a much better job in class and on the written report. Whenever these activities are used as laboratory experiments, it is recommended that students will write laboratory reports to present their data, calculations, and analysis.

The assessment tools appropriate for the mathematics classrooms have been developed with the following in mind: the need to reduce the amount of time that teachers will spend grading the lab reports, and at the same time help students to learn how to write a laboratory report. The assessment of an experiment includes two parts. The first part is pre-lab Performance Based Assessment [Lyublinskaya, 2003a&b]. It includes set of questions with scoring rubrics that students should be able to answer **before** they start an experiment. In many cases, that also means that students are expected to complete necessary calculations prior to the data collection. The performance based assessment form is provided to students at the time when the laboratory experiment is assigned. The teacher has the option to use this form for students' self-evaluation, peer evaluation, or the teacher may use it to interview students before or during the experiment.

The second part of the assessment is the checklist for the written laboratory report. All students should know the requirements for the laboratory reports before they turn them in. The Assessment of Laboratory Report form [Lyublinskaya, 2003a&b] is designed to provide students with the checklist/criteria that they can use when preparing written reports upon completion of the lab. Students use this form for self and peer evaluation, and it also becomes a learning tool for them. Each student is expected to check his/her laboratory report against this checklist. When working as a group, the peer review is also required. This peer evaluation does not include grading by the peers or evaluation of the contribution made by each member of the team. The purpose of the peer evaluation is to make sure that each member of the lab group goes through the completed lab report and checks it against the criteria, makes comments and suggestions for the lab report, revises and perfects their work before it is turned in to the teacher. Each student (or group) is required to turn in an original draft of the lab report, to use the checklist to assess individual or group draft of the lab report, and turn in a final revised copy. The teacher then assesses the final copy of the lab report using the same form. This three-step evaluation allows the teacher to encourage students to check their work before turning it in. It also allows students to learn from each other, and to succeed in lab report writing. At the same time, standardized expectations force students to develop a uniform structure of the lab reports while self and peer evaluation reduces the amount of careless mistakes and omissions in the lab report. This entire process reduces the teacher's grading time.

One more concern regarding the assessment of laboratory experiments (or any group projects) is how to assess individuals within a group. There are different approaches to the group assessment. The laboratory assessment options of either working as a group or individually are offered to the students due to the need to ensure that all students have a clear understanding of the material covered within each laboratory/project, and to ensure a level of equity in the distribution of work. In this approach students choose Laboratory Assessment Options [Lyublinskaya, 2003a&b] that better fit their needs and ability to work within a group. Students are expected to make the choice of an option before they turn in lab reports. My experience shows that about 70% of students usually choose the 1st option, working together and submitting one lab report per group, while 30% of students choose the 2nd option, working individually on the lab report and limit group work to experimentation only. There are a lot of factors that could affect students' choice of the 1st or the 2nd option.

Conclusion

Experiments such as those described above are intended as supplementary activities. Any activity can be used as a teacher's demonstration, class exercise, or laboratory assignment. Using an experiment as a demonstration allows a teacher to talk about real-life applications of mathematics without the necessity to have multiple sets of equipment for the student groups. When activities are used as class exercise or laboratory assignments, students have an opportunity to work as a team and interact with each other. They also acquire learning skills related to the use of measuring devices; however, any group work is more time consuming and usually requires at least one class period and additional time for pre-lab calculations and/or post-lab analysis. Some experiments may be divided into parts and completed within two or three lessons. The teacher may use one part of the experiment as a class demonstration and another part as a lab exercise.

In teaching a particular topic, the teacher has an opportunity to introduce the experimental activities when she or he deems it most appropriate. Labs could be a great way to explore and introduce a new topic that would be followed by the teacher's instructions and explanations. The experiments could also be used as a review exercise. In some cases experiments allow for a more engaging way to exercise the skills necessary for successful learning of mathematics. The most common placement of lab experiments is at the end of a studied topic when students are expected to apply what they learned.

Whatever place the experiments are used within the context, they can enhance students' learning of the mathematics, allow students to see real-life applications and allow the teacher to have performance-based assessments of students' understanding of learned material.

References.

Cortes-Figueroa, J. E., & Moore, D. A. (1999). Using CBL technology and a graphing calculator to teach the kinetics of consecutive first-order reactions. *Journal of Chemical Education,* 76(5), 635-638.

Lyublinskaya, I. (2003a) Connecting Mathematics with Science: Experiments for Precalculus. Emeryville, CA: Key Curriculum Press.

Lyublinskaya, I. (2003b) Connecting Mathematics and Science: Experiments for Calculus. Emeryville, CA: Key Curriculum Press.

Niess, M. (2001). A Model for Integrating Technology in Preservice Science and Mathematics Content-Specific Teacher Preparation, *School Science and Mathematics,* 101 (2), 102-109.

PASCO Scientific, http://pasco.com

Stager, G. (2000). Portable probeware: Science education's next frontier. *Curriculum Administrator*, 36(3), 32-36.

Vernier Software and Technology, http://www.vernier.com

Texas Instruments, http://education.ti.com